

JOHN C. LINDHOLM

A VARIATION OF PARAMETER TECHNIQUE FOR TORQUE ERROR
MINIMIZATION FOR A FOUR BAR MECHANISM WITH A
DESIRED INPUT-OUTPUT TORQUE RELATIONSHIP

by

GARY F. MUELLER

B. S., University of Missouri at Rolla, 1966

A MASTER'S REPORT

submitted in partial fulfillment of the
requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

Approved by:


Major Professor

LD
2668
R4
1968
m8
C.2

A11206 938545

PREFACE

The coordinated motion four bar linkage is a fairly common type of mechanism. It has many applications. It is the purpose of this report to present a method for improving graphical overlay designs by decreasing their input-output torque error. It is also the purpose of this report to present a method of analysis which is more accurate and gives a better indication over the range of operation of the error between the torque required to make a four bar mechanism static, and the torque produced by an external device used to put the mechanism in static equilibrium.

Indebtedness is acknowledged to Dr. John C. Lindholm for his suggestion for work in this area. Also, I would like to thank Dr. Hugh Walker for his advice on computer programming.

TABLE OF CONTENTS

Chapter	Page
I. INTRODUCTION.....	1
The Graphical Overlay Method.....	1
The Variation of Parameter Technique.....	7
II. DEFINITION OF THE PROBLEM.....	10
III. A SPECIFIC PROBLEM.....	11
IV. THE COMPUTER PROGRAM AND ITS USE.....	20
V. SUMMARY AND CONCLUSIONS.....	23
SELECTED BIBLIOGRAPHY.....	24
APPENDIX.....	25
A. Equilibrium Equations.....	26
B. Geometry.....	31
C. Computer Program.....	34

LIST OF TABLES

Table	Page
1. Schedule of Positions for the Example Problem.....	14
2. Computer Printout for the Example Problem.....	16

LIST OF FIGURES

Figure	Page
1. Layout of Successive Positions for the First Crank.....	2
2. Successive Positions for Crank One and Loci of Positions for Extreme End of Connecting Rod.....	3
3. Second Layout with Positions of Crank Two Corresponding to Required Positions.....	5
4. Overlay of Layouts to Determine Length of Crank Two and Link One...	6
5. Error for a Particular Link With All Other Parameters Held Constant.....	8
6. General Picture of the Example Problem.....	12
7. A Graphical Solution for the Example Problem.....	15

CHAPTER I

INTRODUCTION

Often it is a necessity to design a four-bar linkage for which the input and output links perform in some predetermined fashion. The criteria for excellence of performance is the absolute value of the magnitude of error with respect to the desired input-output relationship.

The problem considered is one for which the desired relationship, which may be determined from external torque requirements, involves producing coordinated input-output crank motions.

A graphical method for design of a four-bar mechanism producing angular displacements according to an input-output schedule is available and quite useful. Seeking to reproduce the graphical method on the computer led to a variation of parameter technique which aids the linkage designer multifold.

The Graphical Overlay Method

In order to design a four-bar mechanism which will produce a specified output angle for a specified input angle a simple procedure must be followed:

1. Develop a schedule of positions for the specified input angles and the related output angles.
2. On tracing paper make a layout (Figure 1) showing the successive positions of the first crank. Choose any desirable length for this crank. Assume a length for the connecting rod, and draw the family of circular arcs of this radius with their centers at the successive crankpin positions (Figure 2).

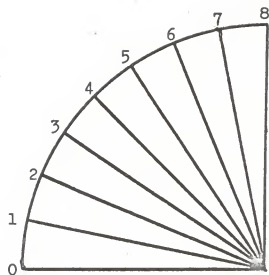


Figure 1, Layout of Successive Positions
for the First Crank

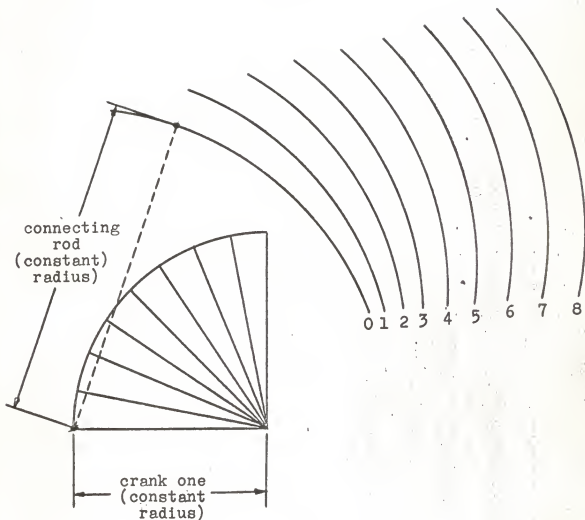


Figure 2, Successive Positions for Crank One and Loci of Positions for Extreme End of Connecting Rod

3. Make a second layout (Figure 3) showing the successive positions of the second crank and a series of possible lengths for this crank.
4. Fit the first layout over the second, as shown in Figure 4, trying to make the circular arcs of the first pass, in proper order, through one of the series of constant radius crankpin positions of the second. It may not be possible to obtain a completely satisfactory fit with the first choice for a connecting rod. If this is the case, the first layout must be redrawn with a different choice of length for the connecting rod, [1]* pp. 45-46.

It is generally accepted that the accuracy of such a method is in the order of $\pm .5$ degrees. The only method for improving accuracy is to construct larger overlays containing more intermediate positions. Calculation of the error with respect to the desired relationship is extremely tedious and errors in calculation become cumulative. The end result is a calculated value of error which may have very little significance. Since nothing is known about the error between the positions chosen for the overlays, these calculations may not even indicate the true value of maximum error.

The graphical method offers no control over link lengths and it often becomes awkward trying to satisfy as many intermediate positions as possible over a specified range.

Two good features of the graphical method are that it contains no complicated theory and it is easy to construct the overlays which will produce fairly good results. Essentially, the graphical method varies three parameters simultaneously. They are: link one, link four, and the initial position.

*See Selected Bibliography

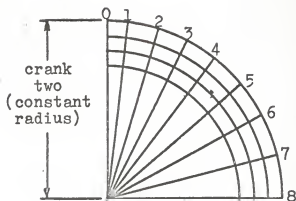


Figure 3, Second Layout with Positions of Crank Two Corresponding to Required Positions

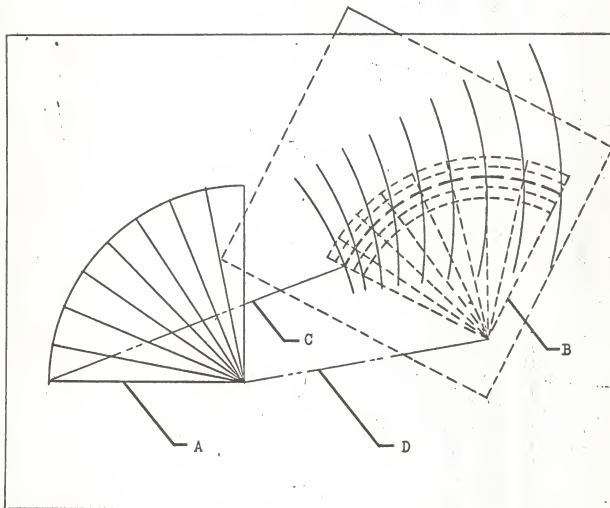


Figure 4, Overlay of Layouts to Determine Length of Crank Two and Link One

- A. Crank One (Link Two)
- B. Crank Two (Link Four)
- C. Connecting Rod (Link Three)
- D. Distance Between Centers (Link One)

The designer's eye is used as the feedback mechanism to produce least error. Should the design appear to be unsatisfactory the designer changes the value of link three and repeats the graphical overlay procedures.

The Variation of Parameter Technique

There are five important parameters associated with working a coordinated motion four bar linkage problem. They are the lengths of the links one through four and the initial position of the mechanism (see Figure B-1 in Appendix B).

After having drawn several graphical solutions, it was observed that the chances for obtaining a completely satisfactory mechanism on the first trial were roughly 50 percent. No positive evidence was given by the graphical process to support a claim that a better solution could be obtained only by increasing or decreasing the value of some parameter.

It seems heuristically evident that if a solution which is close to having minimum error is obtained, a parameter at a time may be varied over a range containing the base solution; and if the parameter variation which produces the least absolute value of maximum error is retained as a new base, a minimum error will be approached. While it is true that the best path of variation may not have been chosen (if the error is dependent on path) a better than initial mechanism is always produced.

If the absolute value of error versus a particular parameter (link length or initial position) is graphed continuously for a range of values containing the base design, the validity of the argument can be easily seen. Figure 5 shows a hypothetical case for which the absolute value of error is plotted against a range of a particular link.

In this example the error is calculated for nine link lengths. One

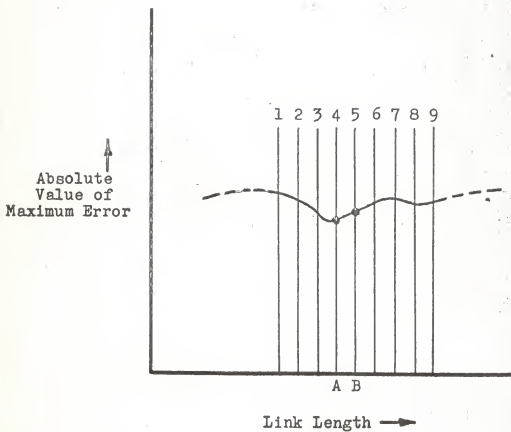


Figure 5, Error for a Particular Link With All Other Parameters Held Constant

- A. Value of Link Chosen as a Minimum
- B. Base Design Link Length

of these calculations must be the base design. The minimum calculated value of maximum absolute error is for link value A. It should be noted that the error at length A may not be the absolute minimum in the range.

As the method directs, some other parameter is optimized in the same fashion. If the maximum absolute value of error versus the same link for which the error was first minimized is graphed, it is noted that the new absolute value of error for link length A is either as small or smaller than the minimum error due to the first parameter optimization.

The steps for solving the problem after the desired relationship between input and output crank is established are as follows:

1. Develop a graphical solution.
2. Choose a small range of values, containing the base design values, in which some or all of the parameters are to be varied. The designer may choose not to vary some parameter.
3. Vary a single parameter at a time over its entire range. The other parameters are constants during this operation.
4. Retain the parameter value which renders the least absolute value of maximum error as a new base.
5. Vary any other parameter as prescribed in 3 and 4.
6. Repeat the process until the error can no longer be reduced or a satisfactory result is obtained.

The most evident advantage of the method is the designer has full control over as many parameters as he wishes to control. If done on the computer, the process can be made to compute the error at many intermediate positions, with tremendous speed, and great accuracy. Thus, more can be learned about the original graphical design and its subsequent improvements. The program for doing this is discussed in Chapter IV.

CHAPTER II

DEFINITION OF THE PROBLEM

To further define the type of problem considered, it would be wise at this time to state some assumptions. They are:

1. All links are rigid.
2. All links are weightless.
3. The linkage is static.
4. All solutions are limited (due to complexity of geometry) to the region above the infinite line through link one's centerline.
5. Link two plus link three plus link four must be greater than link one.
6. Only pure torques are applied to the mechanism. They are on link two and link four.
7. The pins are frictionless in the analysis.
8. Neither of the cranks can reverse direction over the prescribed range.
9. Both cranks travel either counterclockwise or clockwise.

With these assumptions it is possible to analyze the four bar extensively.

An analysis is presented in Appendix A.

If there is an input-output torque relationship which must be maintained, one torque function may be applied to either the input or output crank and the parameters may be adjusted so that the balancing torque on the other crank approaches the second torque function over a specified range.

CHAPTER III

A SPECIFIC PROBLEM

To further illustrate the variation of parameter process, consider the following problem, which is problem E3.5 in Hall's text [1].

Design a spring-linkage counterbalancing system of the type shown in Figure 6. The weight, W , is 50 lb and the pulley radius is 12 in. It is required that the weight be counterbalanced over a 90-deg. range of pulley rotation, e.g., within this range the weight should remain in any position in which it is placed. Neglecting friction, this would mean that the force exerted by link 3 on link 4 should have a moment about the pivot just equal to the moment WR . However, it is anticipated that there will be enough friction in the joints so that it will be necessary to balance the moment WR only within $\pm 3\%$.

The spring is assumed to be linear and is to be assembled so that the spring torque (exerted on link 2) will range from T_1 to $2T_1$ over the counterbalancing range. Design for a 90-deg. range of motion for link 2.

Solving for the spring constant (k):

$$k = (2T_1 - T_1)/\Delta\theta = T_1/\Delta\theta = T_1/(\pi/2) = 2T_1/\pi.$$

The torque in the spring is:

$$T = T_1 + \theta k.$$

The potential energy lost due to a downward displacement of the weight is stored in the helical spring. By using an energy balance, the initial torque, T_1 , and the functional relationship between input and output displacement angles is established.

The energy balance is:

$$\int_0^{\pi/2} (T_1 + \theta k) d\theta = \int_0^{\pi/2} WR d\theta.$$

Solving for T_1 :

$$T_1 = 100/3 \text{ foot pounds.}$$

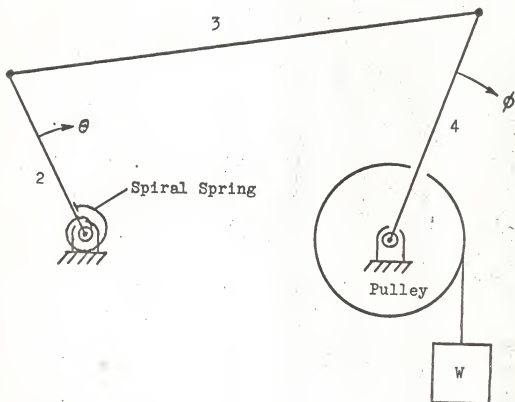


Figure 6, General Picture of the
Example Problem [1]

Solving the energy equation for some intermediate position θ_n and ϕ_n :

$$\int_0^{\theta_n} (T_1 + \theta k) d\theta = \int_0^{\phi_n} WR d\phi;$$

$$T_1 \theta_n + k \theta_n^2 / 2 = WR \phi_n.$$

From this equation a table for input and output angular displacements was developed, Table 1. A graphical solution was obtained. It is displayed as Figure 7. Note that step one has just been completed. (pg)

The length of link four and the initial position were the parameters chosen to vary. The values and ranges for these parameters were:

Link four = $2.06 \pm .06$ units;

output angle = $.6455 \pm .008$ radians.

This completed step two.

Starting with the smallest value, link four was positively incremented until step four was complete. The output angle was incremented similarly. For this problem one cycle of these two parameter variations was sufficient to illustrate the process.

The program for doing this is in Appendix C. An increment size of .020 units was chosen for link four. An increment size of .002 radians was chosen for the initial output angle. The computer printout for the problem is shown in Table 2.

As the computer incremented the value of link four through its range, it not only calculated the error for the graphical design, but also computed the error for a link four length that had less error than the graphical design.

The best linkage in the range was when link four equaled 2.02 units. The maximum error was computed to be 7.857 percent for this link length. The graphical design had a maximum error of 10.99 percent. This was an improvement of 3.033 percent due to only one parameter variation process. Since the

TABLE 1
SCHEDULE OF POSITIONS FOR THE EXAMPLE PROBLEM

Position	θ (degrees)	θ (radians)	ϕ (degrees)	ϕ (radians)
0	0.00	0.0	0.0	0.
1	11.25	$\pi/16$	6.99	0.1219
2	22.50	$\pi/8$	16.90	0.2945
3	33.75	$3\pi/16$	26.65	0.4660
4	45.00	$\pi/4$	37.45	0.6540
5	56.25	$5\pi/16$	49.20	0.8590
6	67.50	$3\pi/8$	61.90	1.0800
7	78.75	$7\pi/16$	75.50	1.3180
8	90.00	$\pi/2$	90.00	1.5710

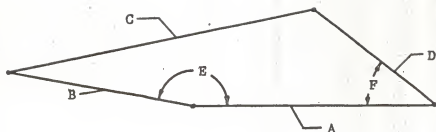


Figure 7, A Graphical Solution for the Example Problem

- A. Link One = 3.19
- B. Link Two = 2.50
- C. Link Three = 4.085
- D. Link Four = 2.06
- E. Initial Angle $(A + B) = 2.9693$ radian
- F. Initial Output Angle = .6455 radian

TABLE 2

COMPUTER PRINTOUT FOR THE EXAMPLE PROBLEM

LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20000000E 01
 INITIAL INPUT ANGLE= 0.29130350E 01
 INITIAL OUTPUT ANGLE= 0.64550000E 00
 MAXIMUM ERROR= 0.97238420E-01

LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 INITIAL INPUT ANGLE= 0.29275450E 01
 INITIAL OUTPUT ANGLE= 0.64550000E 00
 MAXIMUM ERROR= 0.78575780E-01

NOT LEAST ERROR WHEN LINK 4= 0.20399990E 01
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20399990E 01
 MAXIMUM ERROR= 0.85260800E-01

NOT LEAST ERROR WHEN LINK 4= 0.20599990E 01
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20599990E 01
 MAXIMUM ERROR= 0.10992690E 00

NOT LEAST ERROR WHEN LINK 4= 0.20799990E 01
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20799990E 01
 MAXIMUM ERROR= 0.13346610E 00

NOT LEAST ERROR WHEN LINK 4= 0.20999990E 01
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20999990E 01
 MAXIMUM ERROR= 0.15591940E 00

NOT LEAST ERROR WHEN LINK 4= 0.21199990E 01
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.21199990E 01
 MAXIMUM ERROR= 0.17733370E 00

LEAST ERROR DUE TO LINK 4

LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 INITIAL INPUT ANGLE= 0.29275450E 01
 INITIAL OUTPUT ANGLE= 0.64350000E 00
 MAXIMUM ERROR= 0.78575780E-01

LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 INITIAL INPUT ANGLE= 0.29394910E 01
 INITIAL OUTPUT ANGLE= 0.63749990E 00
 MAXIMUM ERROR= 0.75980120E-01

LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 INITIAL INPUT ANGLE= 0.29364900E 01
 INITIAL OUTPUT ANGLE= 0.63949990E 00
 MAXIMUM ERROR= 0.73909870E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.64149990E 00
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 MAXIMUM ERROR= 0.75470860E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.64349990E 00
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 MAXIMUM ERROR= 0.77025890E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.64549990E 00
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 MAXIMUM ERROR= 0.78575780E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.64749990E 00
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 MAXIMUM ERROR= 0.80118230E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.64949990E 00
 LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
 LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
 MAXIMUM ERROR= 0.81697400E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.65149990E 00
LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
MAXIMUM ERROR= 0.83317330E-01

NOT LEAST ERROR WHEN OUTPUT ANGLE= 0.65349990E 00
LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
MAXIMUM ERROR= 0.84927490E-01

LEAST ERROR DUE TO OUTPUT ANGLE
LINK 1= 0.31899990E 01 LINK 2= 0.25000000E 01
LINK 3= 0.40850000E 01 LINK 4= 0.20199990E 01
INITIAL INPUT ANGLE= 0.29364900E 01
INITIAL OUTPUT ANGLE= 0.63949990E 00
MAXIMUM ERROR= 0.73909870E-01

maximum error was less when link four equaled 2.02, this value was retained as the new length for link four.

Next, the output angle was varied as prescribed. The least maximum error for this variation process was found to be 7.391 percent when the initial output angle equaled .6395 radians. This represents an overall reduction of the maximum error of 3.599 percent and .466 percent reduction of the previous least maximum error.

Further similar parameter variations will decrease the error even more. The variation of these two parameters was executed in 48.78 seconds on the IBM 360/50 computer.

This program produced 16 parameter combinations. For each parameter combination the error was computed at every two degrees over a 90 degree displacement of the output angle.

CHAPTER IV

THE PROGRAM AND ITS USE

It is assumed that the designer has at least a cursory knowledge of Fortran programming techniques. This particular program was compiled and executed on the IBM 360/50 computer. The IBM 360/50 accepts cards punched on an IBM 29 Card Punch.

The program is essentially divided into four sections. They are:

1. a data section,
2. a variation of link four and a variation of initial output angle section,
3. an input torque subroutine section, and
4. two \sin^{-1} (argument) functions.

The data section is a group of cards immediately following the format cards. The symbols and their interpretations are as follows:

1. XLINC is the value of the increment size for the link length. It may be any value such that when it is multiplied by some integer and is added to CXI4, the base design value for the linkage is obtained.
2. AINC is the value of the increment size for the initial output angle. It is measured in radians. It may have any value such that when it is multiplied by some integer and is added to AANG, BANG is obtained.
3. AANG is a value of the initial output angle which is smaller than the graphical design value. It is measured in radians. It must

- have a value such that an integer times AINC plus AANG equals BANG.
4. BANG is the graphical design value for the initial output angle.
It is measured in radians.
 5. XL1 is link one.
 6. XL2 is link two.
 7. XL3 is link three.
 8. CXL4 is a constant value for link four. It is smaller than the graphical design value for link four and has a value such that when it is added to the product of an integer and XLINC, the graphical value of link four is obtained.
 9. SK is the spring constant. Omit this card if there is no spring on the input crank.
 10. TI is the initial torque on the spring. Omit this card if there is no initial torque.

The variation of link four and initial output angle section follows the data section. The output angle is incremented in $2\pi/180$ radian steps over a $\pi/2$ radian displacement. This may easily be changed.

In order to change the increment size of the output angle simply change the card in this section incrementing the output angle from $XOUTA = XOUTA + 2.*3.141592/180.$ to $XOUTA = XOUTA + \text{desired increment size in radians}.$ This output angle displacement divided by the desired increment size must be an integer (N). The DO loop index must be changed to $I = 1, N + 1.$

The torque the designer is trying to simulate is the torque developed in a spiral spring (TSP). The designer may wish to use another device on the input crank. This will necessitate the replacement of the TSP card by an appropriate function. Also, the error (A5) card must be repunched to conform

to the new expression for error.

If it is desired to vary more parameters, the inserted cards follow a pattern similar to the pattern established by the first two parameter variation processes.

The torque subroutine calculates an input torque for a constant output torque. The output torque may be easily changed from a constant torque to a torque with a value dependent on the output angle by replacing the card marked $TOUT = 50.$ to $TOUT = f(XOUTA).$

Since there is a continuous need to solve for angles in radians, functions $ANGLE$ and $GANGL$ were developed, $ANGLE$ may be used for angles from 0 to 90 degrees. $GANGL$ is used for angles from 90 to 180 degrees. To use either of these functions simply submit the sine of an angle to the appropriate function and the value of the angle will be calculated and returned to the main program.

CHAPTER V

SUMMARY AND CONCLUSIONS

The steps involved in using the variation of parameter technique are outlined clearly in Chapter I. The steps are simple to follow and are aided greatly by the computer program listed in Appendix C. The use of this program is described in Chapter IV.

It was demonstrated that the error can be significantly decreased by applying the variation of parameter technique. In the example problem the graphical error was 10.99 percent. This error was reduced to 7.391 percent after applying the variation of parameter technique first, to link four, and second, to the initial output angle. Further applications of the technique will reduce the error even more.

These two applications were executed in 48.78 seconds on the IBM 360/50 computer. This represents 46 calculations of the error for each of 16 parameter combinations. Because the error was calculated in many intermediate positions, the maximum error the linkage produced was accurately determined.

The number of intermediate positions can be increased and the size of parameter incrementation can be decreased to produce an even better analysis.

From the results of the example problem it is concluded that the variation of parameter technique can be employed to reduce error and yield a design which is more satisfactory than the design produced by graphical method.

SELECTED BIBLIOGRAPHY

1. Hall, Allen S., Jr. Kinematics and Linkage Design. New Jersey: Prentice-Hall, Inc., 1961.
2. Holowenko, A. R. Dynamics of Machinery. New York, London: John Wiley and Sons, Inc., 1963.

APPENDIX

Appendix A

Equilibrium Equations

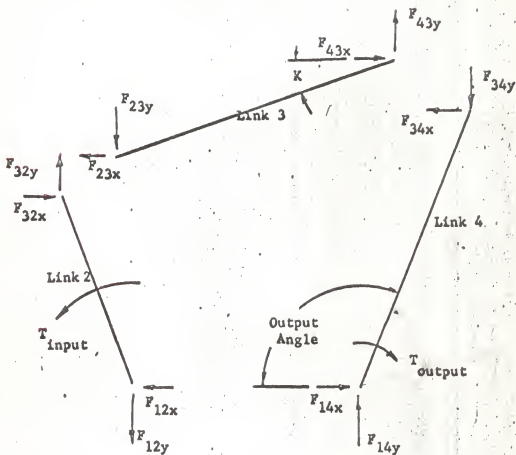


Figure A-1, Torque and Force Components on a Separated Four Bar Mechanism

Separating the mechanism at its pins as in Figure A-1; the force

balance for each link is:

$$F_{14x} - F_{34x} = 0;$$

$$F_{34y} - F_{14y} = 0;$$

$$F_{43x} - F_{23x} = 0;$$

$$F_{43y} - F_{23y} = 0;$$

$$F_{32x} - F_{12x} = 0;$$

$$F_{32y} - F_{12y} = 0.$$

The force balance for each pin is:

$$F_{34x} = F_{43x};$$

$$F_{34y} = F_{43y};$$

$$F_{23y} = F_{32y};$$

$$F_{23x} = F_{32x};$$

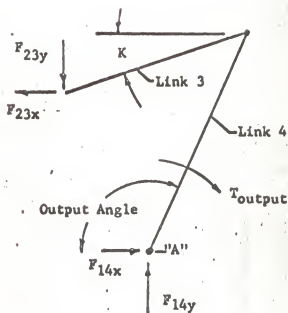


Figure A-2, Forces and Torque on Links Three and Four

Performing a balance of torques about point "A" for Figure A-2 obtains:

$$T_{\text{output}} - F_{23x}[(\text{Link } 4)(\sin(\text{Output Angle})) - (\text{Link } 3)(\sin(K))]$$

$$- F_{23y}[(\text{Link } 4)(\cos(\text{Output Angle})) + (\text{Link } 3)(\cos(K))] = 0.$$

[1]

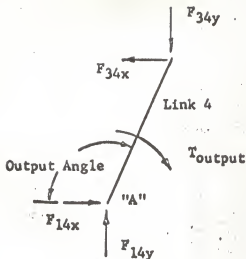


Figure A-3, Forces and Torque on Link 4

Performing a balance of torques about point "A" for Figure A-3, and realizing $F_{34x} = F_{23x}$ and $F_{34y} = F_{23y}$ obtains:

$$T_{\text{output}} - F_{23x}(\text{Link } 4)(\sin(\text{Output Angle})) - F_{23y}(\text{Link } 4)(\cos(\text{Output Angle})) = 0.$$

Let:

$$AA = (\text{Link } 4)(\sin(\text{Output Angle})),$$

$$BB = (\text{Link } 3)(\sin(K)),$$

$$CC = (\text{Link } 4)(\cos(\text{Output Angle})),$$

$$DD = (\text{Link } 3)(\cos(K)).$$

Equation [1] becomes:

$$F_{23x}(AA - BB) + F_{23y}(CC + DD) = T_{\text{output}},$$

Equation [2] becomes:

$$F_{23x}(AA) + F_{23y}(CC) = T_{\text{output}}.$$

Solving for F_{23x} and F_{23y} :

$$F_{23x} = \frac{\begin{vmatrix} T_{\text{output}} & CC + DD \\ T_{\text{output}} & CC \end{vmatrix}}{\begin{vmatrix} AA - BB & CC + DD \\ AA & CC \end{vmatrix}} = \frac{T_{\text{output}} (DD)}{(BB)(CC) + (AA)(DD)} \quad [5]$$

$$F_{23y} = \frac{\begin{vmatrix} AA - BB & T_{\text{output}} \\ AA & T_{\text{output}} \end{vmatrix}}{\begin{vmatrix} AA - BB & CC + DD \\ AA & CC \end{vmatrix}} = \frac{T_{\text{output}} (BB)}{(BB)(CC) + (AA)(DD)} \quad [6]$$

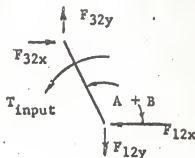


Figure A-4, Forces and Torque on Link 2

Performing a balance of torques about point "B" for Figure A-4, and realizing $F_{32x} = F_{23x}$ and $F_{32y} = F_{23y}$ yields:

$$T_{\text{input}} = F_{23x} (\text{Link } 2) (\sin(A + B)) - F_{23y} (\cos(A + B)).$$

It is possible at this point to solve for T_{input} for any T_{output} .

Appendix B

Geometry

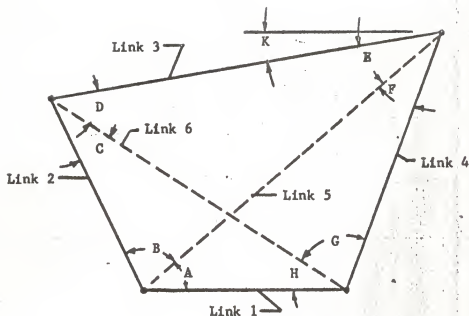


Figure B-1, Geometry of a Four Bar Linkage

If all the links and $G + H$ are known:

$$\text{Link 5} = \sqrt{(\text{Link 4})^2 + (\text{Link 1})^2 - 2(\text{Link 1})(\text{Link 4})\cos(G + H)}$$

$$A = \sin^{-1} \left[\frac{\sin(G + H)(\text{Link 4})}{\text{Link 5}} \right]$$

$$C + D = \cos^{-1} \left[\frac{\sqrt{(\text{Link 5})^2 - (\text{Link 2})^2 - (\text{Link 3})^2}}{\sqrt{2}(\text{Link 3})(\text{Link 2})} \right]$$

$$B = \sin^{-1} \left[\frac{\sin(C + D)(\text{Link 3})}{\text{Link 5}} \right]$$

$$E = \sin^{-1} \left[\frac{\sin(C + D)(\text{Link 2})}{\text{Link 5}} \right]$$

$$F = \sin^{-1} \left[\frac{\sin(G + H)(\text{Link 1})}{\text{Link 5}} \right]$$

$$\text{Link 6} = \sqrt{(\text{Link 1})^2 + (\text{Link 2})^2 - 2(\text{Link 1})(\text{Link 2})\cos(A + B)}$$

$$G = \sin^{-1} \left[\frac{\sin(E + F)(\text{Link 3})}{\text{Link 6}} \right]$$

$$H = \sin^{-1} \left[\frac{\sin(A + B)(\text{Link 2})}{\text{Link 6}} \right]$$

$$C = \sin^{-1} \left[\frac{\sin(H)(\text{Link 1})}{\text{Link 2}} \right]$$

$$D = \sin^{-1} \left[\frac{\sin (E+F) (\text{Link } 4) / \text{Link } 6}{} \right]$$

$$K = 180^\circ - (G+H) - (E+F)$$

Appendix C

Computer Program

```

9000 FORMAT(8H LINK 1=,E16.8,2X,8H LINK 2=,E16.8,/,8H LINK 3=,E16.
      12X,6H LINK 4=,E16.8,/,21H INITIAL INPUT ANGLE=,E16.8,/,
      222H INITIAL OUTPUT ANGLE=,E16.8,/,15H MAXIMUM ERROR=,E16.8,/)
9002 FORMAT(6H LINK 1=,E16.8,2X,8H LINK 2=,E16.8,/,8H LINK 3=,E16.
      12X,8H LINK 4=,E16.8,/,15H MAXIMUM ERROR=,E16.8,/)
9003 FORMAT(26H LEAST ERROR DUE TO LINK 4)
9004 FORMAT(17H VOID SET LINK 4=,E16.8,/)
9005 FORMAT(29H NOT LEAST ERROR WHEN LINK 4=,E16.8)
9007 FORMAT(35H NOT LEAST ERROR WHEN OUTPUT ANGLE=,E16.8)
9008 FORMAT(23H VOID SET OUTPUT ANGLE=,E16.8,/)
9009 FORMAT(32H LEAST ERROR DUE TO OUTPUT ANGLE)
      XLINC=.020
      AINC=.002
      ARIA=0.
      AANG=.6375
      RANG=.6455
      XL1=3.19
      XL2=2.50
      XL3=4.085
      CXL4=2.00
      XL4=CXL4
      COU=0.
      1 SK=200./(3.*3.141592)
      2 TI=100./3.
      RNO=2000.
      UNO2=2000.
100 XOUTA=UANG
101 XERR=0.
      COU=COU+1.
      CALL TORQ (XL1,XL2,XL3,XL4,XOUTA,TIN,CSB,CSA,SNAB,CSAB,E,F,
      1SNA,SNB,XKR,F23X,F23Y,A,B)
17 ZERO=A+B
      ZOUTA=XOUTA
      DO 300 I=1,46
      CALL TORQ (XL1,XL2,XL3,XL4,XOUTA,TIN,CSB,CSA,SNAB,CSAB,E,F,
      1SNA,SNB,XKR,F23X,F23Y,A,B)
      TSP=TI+(ZERO-A-B)*SK
      A5=(TIN-TSP)/TSP
      ERR=ABS(A5)
      IF(ERR-XERR)201,201,203
203 XERR=ERR
201 CONTINUE
      XOUTA=XOUTA+2.*3.141592/180.
      IF(ABA-1.)299,304,304
299 IF(XOUTA-3.141592)202,202,301
202 AQ=(XL2+XL3)*(XL2+XL3)
      AR=XL1*XL1+XL4*XL4-2.*XL1*XL4*COS(XOUTA)
      IF(AQ-AR)301,301,300
304 IF(XOUTA-3.141592)305,305,401
305 AQ=(XL2+XL3)*(XL2+XL3)
      AR=XL1*XL1+XL4*XL4-2.*XL1*XL4*COS(XOUTA)
      IF(AQ-AR)401,401,300

```

```

300 CONTINUE
    IF (ABA-1.) 204, 400, 400
204 IF (XERR-BNO) 250, 250, 252
250 BNO=XFRR
    AXL1=XL1
    AXL2=XL2
    AXL3=XL3
    AXL4=XL4
    AZERO=ZERO
    AOUTA=ZOUTA
    WRITE(3,9000)AXL1,AXL2,AXL3,AXL4,AZERO,AOUTA,BNO
    GO TO 254
252 WRITE(3,9005)XL4
    WRITE(3,9002)XL1,XL2,XL3,XL4,XERR
    XL4=CXL4+XLINC*COU
    IF (XL4-1.06*CXL4) 302, 302, 303
302 GO TO 100
301 CONTINUE
253 WRITE(3,9004)XL4
254 XL4=CXL4+COU*XLINC
    IF (XL4-1.06*CXL4) 302, 302, 303
303 WRITE(3,9003)
    WRITE(3,9000)AXL1,AXL2,AXL3,AXL4,AZERO,AOUTA,BNO
    ABA=1.
    XL4=AXL4
    COU=0.
    XOUTA=AANG
    GO TO 101
400 CONTINUE
404 IF (XFRR-BNO2) 450, 450, 452
450 BNO2=XERR
    BXL1=XL1
    BXL2=XL2
    BXL3=XL3
    BXL4=XL4
    BZERO=ZERO
    BOUTA=ZOUTA
    WRITE(3,9000)BXL1,BXL2,BXL3,BXL4,BZERO,BOUTA,BNO2
    GO TO 454
452 WRITE(3,9007)ZOUTA
    WRITE(3,9002)XL1,XL2,XL3,XL4,XERR
    XOUTA=AANG+COU*A INC
    IF (XOUTA-1.013*BANG) 402, 402, 403
402 GO TO 101
401 CONTINUE
453 WRITE(3,9008)ZOUTA
454 XOUTA=AANG+COU*A INC
    IF (XOUTA-1.013*BANG) 402, 402, 403
403 WRITE(3,9009)
    WRITE(3,9000)BXL1,BXL2,BXL3,BXL4,BZERO,BOUTA,BNO2
    STOP
    END

```



```

SUBROUTINE TORQ(XL1,XL2,XL3,XL4,XOUTA,TIN,CSB,CSA,SNAB,CSAB,E,F
1SNA,SNB,XKR,F23X,F23Y,A,B)
1002 AG=(XL4*XL4+XL1*XL1-2.*XL1*XL4*COS(XOUTA))
17 XL5=SQRT(AG)
18 CSNCD=(XL5*XL5-XL3*XL3-XL2*XL2)/(-2.*XL3*XL2)
AH=1.-CSNCD*CSNCD
SNCD=SQRT(AH)
20 SNF=SIN(XOUTA)*XL1/XL5
PP=1.
21 AA=XL4*XL4
22 BB=XL3*XL3
23 CC=XL1*XL1
24 IF(DD+AA-CC)25,26,27
25 Z=SNF
GO TO 499
26 F=3.141592/2.
GO TO 28
27 Z=SNF
GO TO 379
28 SNE=XL2*SNCD/XL5
29 PP=2.
30 DD=XL2*XL2
31 EE=XL3*XL3
32 IF(DD-EE-BB)33,34,35
33 Z=SNE
GO TO 379
34 E=3.141592/2.
GO TO 36
35 Z=SNE
GO TO 499
36 XKR=3.141592-XOUTA-E-F
40 TOUT=50.
AAAH=1./(XL4*COS(XOUTA)+XL3*COS(XKR))
AAAI=1./(XL4*COS(XOUTA))
AAAK=SIN(XOUTA)/COS(XOUTA)
AAAJ=(XL4*SIN(XOUTA)-XL3*SIN(XKR))/(XL4*COS(XOUTA)+XL3*COS(XKR))
42 F23X=-TOUT*(AAAH-AAAI)/(AAAK-AAAJ)
AAAD=1./(XL4*SIN(XOUTA)-XL3*SIN(XKR))
AAAE=1./(XL4*SIN(XOUTA))
AAAF=COS(XOUTA)/SIN(XOUTA)
AAG=(XL4*COS(XOUTA)+XL3*COS(XKR))/(XL4*SIN(XOUTA)-XL3*SIN(XKR))
43 F23Y=-TOUT*(AAAD-AAAE)/(AAAF-AAAG)
SNA=SIN(XOUTA)*XL4/XL5
SNB=SNCD*XL3/XL5
AQ=1.-SNA*SNA
AP=1.-SNB*SNB
IF(AA-CC-BB)44,45,46
44 CSA=SQRT(AQ)
GO TO 47
45 CSA=0.
GO TO 47
46 CSA=-SQRT(AQ)

```

```

47 DD=XL2*XL2
48 IF(DD+DD-EE)49,50,51
49 CSB=-SQRT(AP)
GO TO 52
50 CSB=0.
GO TO 52
51 CSB=SQRT(AP)
52 CSAF=CSA*CSB-SNA*SNB
55 SNAH=SNA*CSB+CSA*SNB
PP=3.
IF(CSA-0.)705,706,707
705 Z=SNA
GO TO 499
706 A=3.141592/2.
GO TO 709
707 Z=SNA
708 GO TO 379
709 PP=4.
IF(CSB-0.)710,711,712
710 Z=SNB
GO TO 499
711 B=3.141592/2.
GO TO 713
712 Z=SNB
GO TO 379
379 IF(Z-.7)380,380,401
380 XN=-1.
381 XX=0.
382 XSUM=Z
373 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
XX=XX+XSUM
398 IF(XSUM-.00001)400,400,399
399 XN=XN+2.
GO TO 383
400 ANGLE=Z+XX
GO TO 600
401 XN=-1.
Z=1.-Z*Z
Z=SQRT(Z)
XX=0.
XSUM=Z
402 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
XX=XX+XSUM
404 IF(XSUM-.00001)405,405,402
405 ANGLE=3.141592/2.-(Z+XX)
GO TO 600
499 IF(Z-.7)500,500,520
500 XN=-1.
XX=0.
XSUM=Z
501 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
XX=XX+XSUM
1909 IF(XSUM-.00001)503,503,502

```

```
502 XN=XN+2.  
GO TO 501  
503 ANGLE=-(Z+XX)+3.141592  
GO TO 600  
520 XN=-1.  
XX=0.  
Z=1.-Z*Z  
Z=SQRT(Z)  
XSUM=Z  
521 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))  
XX=XX+XSUM  
524 IF(XSUM-.00001)525,525,521  
525 ANGLE=3.141592/2.+Z+XX  
600 IF(PP-1.)599,601,603  
601 F=ANGLE  
602 GO TO 28  
603 IF(PP-2.)599,604,606  
599 XOUTA=ANGLE  
604 E=ANGLE  
605 GO TO 36  
606 IF(PP-3.)605,607,609  
607 A=ANGLE  
608 GO TO 709  
609 IF(PP-4.)608,610,612  
610 U=ANGLE  
611 GO TO 713  
612 CONTINUE  
713 CONTINUE  
TIN=F23X*XL2*SNAB-F23Y*XL2*CSAB  
RETURN  
END
```

```

FUNCTION ANGLE(Z)
379 IF(Z-.7)380,380,401
380 XN=-1.
381 XX=0.
382 XSUM=Z
383 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
    XX=XX+XSUM
398 IF(XSUM-.00001)400,400,399
399 XN=XN+2.
    GO TO 383
400 ANGLE=Z+XX
    GO TO 600
401 XN=-1.
    Z=1.-Z*Z
    Z=SQRT(Z)
    XX=0.
    XSUM=Z
402 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
    XX=XX+XSUM
404 IF(XSUM-.00001)405,405,402
405 ANGLE=3.141592/2.-(Z+XX)
600 ANGLE=ANGLE
    RETURN
    END

```

```

FUNCTION GANGL(Z)
499 IF(Z-.7)500,500,520
500 XN=-1.
    XX=0.
    XSUM=Z
501 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
    XX=XX+XSUM
509 IF(XSUM-.00001)503,503,502
502 XN=XN+2.
    GO TO 501
503 ANGLE=-(Z+XX)+3.141592
    GO TO 600
520 XN=-1.
    XX=0.
    Z=1.-Z*Z
    Z=SQRT(Z)
    XSUM=Z
521 XSUM=XSUM*(XN+2.)*(XN+2.)*Z*Z/((XN+3.)*(XN+4.))
    XX=XX+XSUM
524 IF(XSUM-.00001)525,525,521
525 ANGLE=3.141592/2.+Z+XX
600 ANGLE=ANGLE
    RETURN
    END

```

A VARIATION OF PARAMETER TECHNIQUE FOR TORQUE ERROR
MINIMIZATION FOR A FOUR BAR MECHANISM WITH A
DESIRED INPUT-OUTPUT TORQUE RELATIONSHIP

by

GARY F. MUELLER

B. S., University of Missouri at Rolla, 1966

AN ABSTRACT OF A MASTER'S REPORT

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Mechanical Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1968

Approved by:


Major Professor

Name: Gary F. Mueller

Date of Degree: January, 1968

Institution: Kansas State University

Title of Study: A VARIATION OF PARAMETER TECHNIQUE FOR TORQUE ERROR
MINIMIZATION FOR A FOUR BAR MECHANISM WITH A DESIRED INPUT-OUTPUT
TORQUE RELATIONSHIP

Pages in Study: 40

Major Field: Mechanical Engineering

Scope and Method of Study: The coordinated motion linkage problem can be solved by an approximate graphical overlay method. The graphical method's results can be then considerably refined by the application of the variation of parameter technique. An example problem which is included in the report considers a linkage which must satisfy certain external torque requirements. It is analyzed as a coordinated motion problem. The linkage's measure of excellence is the absolute value of the difference of the desired torque of link two minus the actual torque on link two divided by the desired torque on link two. A computer program was developed to perform the variation of parameter technique on the graphical solution. It can be easily modified to fit most coordinated motion problems.

Findings and Conclusions: The results of this report indicate that the variation of parameter technique is a valid and useful method for reducing the error in linkage coordinated motions. The steps for performing the technique are clearly outlined. The included computer program is of great use for application of the technique. The results of the example problem point out that error can be significantly reduced

by use of the technique. The computer was utilized to reduce time and increase accuracy of the calculations. The included program can be modified to make an even better approximation for the calculation of the maximum error.

MAJOR PROFESSOR'S APPROVAL

J. Lindholm